Quadriken

$$Q_t := \left\{ \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in \mathbb{R}^2 \mid 6x_1^2 + 6x_2^2 - 8x_1x_2 + 4x_2 + t = 0 \right\}$$

$$x^T A x + 2a^T x + c = 0$$

$$6x_1^2 + 6x_2^2 - 8x_1x_2 \Rightarrow A = \begin{pmatrix} 6 & -4 \\ -4 & 6 \end{pmatrix}$$

$4x_2 \rightarrow a = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$, $c = t$

$$\chi_A(\lambda) = \det(A - \lambda E_2) = \begin{vmatrix} 6-\lambda & -4 \\ -4 & 6-\lambda \end{vmatrix} = (6-\lambda)^2 - 16$$

$$\chi_A(\lambda) \overset{!}{=} 0 \Rightarrow (6-\lambda)^2 = 4^2 \Rightarrow 6 - \lambda = \pm 4$$

$$\Rightarrow \begin{cases} \lambda_1 = 6 - 4 = 2 \\ \lambda_2 = 6 + 4 = 10 \end{cases}$$

$$\lambda_1 = 2 \Rightarrow (A - 2E_2)x = 0 \Rightarrow \left[\begin{array}{cc|c} 6-2 & -4 & 0 \\ -4 & 6-2 & 0 \end{array}\right] \Rightarrow$$

$$\Rightarrow \left[\begin{array}{cc|c} 4 & -4 & 0 \\ -4 & 4 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 4 & -4 & 0 \\ 4 & -4 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 1 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right] \Rightarrow$$

$$\Rightarrow x_1 - x_2 = 0 \Rightarrow x_1 = x_2 \Rightarrow v_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\lambda_2 = 10 \Rightarrow (A - 10E_2)x = 0 \Rightarrow \left[\begin{array}{cc|c} 6-10 & -4 & 0 \\ -4 & 6-10 & 0 \end{array}\right] \Rightarrow$$

$$\Rightarrow \left[\begin{array}{cc|c} -4 & -4 & 0 \\ -4 & -4 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 4 & 4 & 0 \\ 0 & 0 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right] \Rightarrow$$

$$\Rightarrow x_1 + x_2 = 0 \Rightarrow x_1 = -x_2 \Rightarrow v_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$w_1 = \frac{v_1}{\|v_1\|} = \frac{1}{\sqrt{1^2+1^2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\hat{v}_2 = \frac{v_2}{\|v_2\|} = \frac{1}{\sqrt{1^2+(-1)^2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$F = (0, \hat{v}_1, \hat{v}_2) \quad , F = f(\hat{v}_1, \hat{v}_2)$$

$$\Rightarrow F = \left(0, \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}\right), \quad F = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \in K_F$$

$$\Rightarrow y^T (F^T A F) y + 2 (F^T a)^T y + t = 0$$

$$\Leftrightarrow y^T \left(\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 6 & -4 \\ -4 & 6 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \cdot \frac{1}{\sqrt{2}} \right) y + 2 \left(\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 2 \end{pmatrix} \right)^T y + t = 0$$

$$\Leftrightarrow y^T \left(\frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 2 & 10 \\ 2 & -10 \end{pmatrix} \right) y + 2 \left(\frac{1}{\sqrt{2}} \begin{pmatrix} 2 \\ -2 \end{pmatrix} \right)^T y + t = 0$$

$$\Leftrightarrow y^T \frac{1}{2} \begin{pmatrix} 4 & 0 \\ 0 & 20 \end{pmatrix} y + \begin{pmatrix} 2\sqrt{2} \\ 2\sqrt{2} \end{pmatrix}^T y + t = 0$$

$$\Leftrightarrow (y_1, y_2) \begin{pmatrix} 2 & 0 \\ 0 & 10 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} + \begin{pmatrix} 2\sqrt{2} \\ 2\sqrt{2} \end{pmatrix}^T \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} + t = 0$$

$$\Leftrightarrow 2y_1^2 + 10y_2^2 + 2\sqrt{2}\, y_1 + 2\sqrt{2}\, y_2 + t = 0$$

$$\Leftrightarrow 2y_1^2 + 2\sqrt{2}\, y_1 + 10y_2^2 + 2\sqrt{2}\, y_2 + t = 0$$

$$\Leftrightarrow 2(y_1^2 + \sqrt{2}\, y_1) + 10\left(y_2^2 + \frac{\sqrt{2}}{5} y_2\right) + t = 0$$

$$\Leftrightarrow 2\left(y_1 + \frac{\sqrt{2}}{2}\right)^2 - 2 \cdot \frac{1}{2} + 10\left(y_2 + \frac{\sqrt{2}}{10}\right)^2 + t - 10 \cdot \frac{1}{10} = 0$$

$$\Leftrightarrow 2\left(y_1 + \frac{\sqrt{2}}{2}\right)^2 + 10\left(y_2 + \frac{\sqrt{2}}{10}\right)^2 + t - \frac{6}{5} = 0$$

sei $z_1 = y_1 + \frac{\sqrt{2}}{2}$

$$z_2 = y_2 + \frac{\sqrt{2}}{10}$$

$$\Rightarrow 2z_1^2 + 10z_2^2 + t - \frac{6}{5} = 0$$

Fall 1: $t = \frac{6}{5}$

$\Rightarrow 22z_1^2 + 10z_2^2 = 0$

Fall 2: $t < \frac{6}{5}$

$\Rightarrow 2z_1^2 + 10z_2^2 + (t - \frac{6}{5}) = 0 \quad | \cdot \frac{1}{t - \frac{6}{5}}$

$\Rightarrow \frac{2}{t - \frac{6}{5}} \cdot z_1^2 + \frac{10}{t - \frac{6}{5}} \cdot z_2^2 + 1 = 0$