Koordinatentransformationen

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Aufgabe 1

$G = \left(\left(\begin{pmatrix}2\\1\end{pmatrix}; \frac{1}{\sqrt{5}}\begin{pmatrix}1\\2\end{pmatrix}, \frac{1}{\sqrt{5}}\begin{pmatrix}-2\\1\end{pmatrix}\right)\right)$, ${}_G P = \begin{pmatrix}3\\-1\end{pmatrix}$

a) ${}_E P = ?$

$G = \left(\underbrace{\begin{pmatrix}2\\1\end{pmatrix}}_{g}; \underbrace{\frac{1}{\sqrt{5}}\begin{pmatrix}1\\2\end{pmatrix}, \frac{1}{\sqrt{5}}\begin{pmatrix}-2\\1\end{pmatrix}}_{G}\right)$

$G = \frac{1}{\sqrt{5}}\begin{pmatrix}1 & -2\\2 & 1\end{pmatrix}$ und $g = \begin{pmatrix}2\\1\end{pmatrix}$

${}_E P = G \cdot {}_G P + g$

$\Rightarrow {}_E P = \frac{1}{\sqrt{5}}\begin{pmatrix}1 & -2\\2 & 1\end{pmatrix}\begin{pmatrix}3\\-1\end{pmatrix} + \begin{pmatrix}2\\1\end{pmatrix}$

$= \frac{1}{\sqrt{5}}\begin{pmatrix}1\cdot 3 + (-2)\cdot(-1)\\2\cdot 3 + 1\cdot(-1)\end{pmatrix} + \begin{pmatrix}2\\1\end{pmatrix}$

$= \frac{1}{\sqrt{5}}\begin{pmatrix}5\\5\end{pmatrix} + \begin{pmatrix}2\\1\end{pmatrix}$

$= \begin{pmatrix}\sqrt{5}\\\sqrt{5}\end{pmatrix} + \begin{pmatrix}2\\1\end{pmatrix}$

$= \begin{pmatrix}2+\sqrt{5}\\1+\sqrt{5}\end{pmatrix}$

b) ${}_E K_F(v) = ?$

${}_E K_F(v) = G \cdot v + g$, $v = \begin{pmatrix}v_1\\v_2\end{pmatrix}$

$\Rightarrow {}_G K_F(v) = \frac{1}{\sqrt{5}}\begin{pmatrix}1 & -2\\2 & 1\end{pmatrix} v + \begin{pmatrix}2\\1\end{pmatrix}$

c) ${}_F K_E(v) = ?$

${}_F K_E(v) = \left({}_E K_F(v)\right)^{-1}$

$= G^{-1}(v - g)$

$= G^{-1} v - G^{-1} g$

$\Rightarrow {}_F K_E(v) = \frac{1}{\sqrt{5}}\begin{pmatrix}1 & -2\\2 & 1\end{pmatrix}^{-1} v - \frac{1}{\sqrt{5}}\begin{pmatrix}1 & -2\\2 & 1\end{pmatrix}^{-1}\begin{pmatrix}2\\1\end{pmatrix}$

Bei Abbildungen, $\begin{pmatrix}1 & -2\\2 & 1\end{pmatrix}^{-1}$ berechnet nicht $\frac{1}{\det(A)}\begin{pmatrix}d & -b\\-c & a\end{pmatrix}$, sondern $A^T = \begin{pmatrix}1 & 2\\-2 & 1\end{pmatrix}$

$\Rightarrow {}_F K_E(v) = \frac{1}{\sqrt{5}}\begin{pmatrix}1 & 2\\-2 & 1\end{pmatrix} v - \frac{1}{\sqrt{5}}\begin{pmatrix}1 & 2\\-2 & 1\end{pmatrix}\begin{pmatrix}2\\1\end{pmatrix}$

$= \frac{1}{\sqrt{5}}\begin{pmatrix}1 & 2\\-2 & 1\end{pmatrix} v - \frac{1}{\sqrt{5}}\begin{pmatrix}0\\-3\end{pmatrix}$

Aufgabe 2

$F = \left(\left(\begin{pmatrix}-1\\2\end{pmatrix}; \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}, \frac{1}{\sqrt{2}}\begin{pmatrix}-1\\1\end{pmatrix}\right)\right)$

und ${}_F P = \begin{pmatrix}2\\4\end{pmatrix}$

a) ${}_E P = F \cdot {}_F P + f$

$F = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & -1\\1 & 1\end{pmatrix}$, $f = \begin{pmatrix}-1\\2\end{pmatrix}$

$\Rightarrow {}_E P = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & -1\\1 & 1\end{pmatrix}\begin{pmatrix}2\\4\end{pmatrix} + \begin{pmatrix}-1\\2\end{pmatrix}$

$= \frac{1}{\sqrt{2}}\begin{pmatrix}-2\\6\end{pmatrix} + \begin{pmatrix}-1\\2\end{pmatrix}$

$= \begin{pmatrix}-1-\sqrt{2}\\2+3\sqrt{2}\end{pmatrix}$

b) ${}_F K_E(v) = F v + f$

$= \frac{1}{\sqrt{2}}\begin{pmatrix}1 & -1\\1 & 1\end{pmatrix} v + \begin{pmatrix}-1\\2\end{pmatrix}$

c) ${}_E K_F(v) = F^{-1}(v - f)$

$= F^{-1} v - F^{-1} f$

$\Rightarrow {}_E K_F(v) = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\-1 & 1\end{pmatrix} v - \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\-1 & 1\end{pmatrix}\begin{pmatrix}-1\\2\end{pmatrix}$

$= \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\-1 & 1\end{pmatrix} v - \frac{1}{\sqrt{2}}\begin{pmatrix}1\\3\end{pmatrix}$

Aufgabe 3

$O = \left(\left(\begin{pmatrix}2\\1\end{pmatrix}; \frac{1}{\sqrt{5}}\begin{pmatrix}2\\1\end{pmatrix}, \frac{1}{\sqrt{5}}\begin{pmatrix}-1\\2\end{pmatrix}\right)\right)$, ${}_O P = \begin{pmatrix}-2\\3\end{pmatrix}$

$O = \frac{1}{\sqrt{5}}\begin{pmatrix}2 & -1\\1 & 2\end{pmatrix}$, $\theta = \begin{pmatrix}2\\1\end{pmatrix}$

a) ${}_E P = O \cdot {}_O P + \theta$

$= \frac{1}{\sqrt{5}}\begin{pmatrix}2 & -1\\1 & 2\end{pmatrix}\begin{pmatrix}-2\\3\end{pmatrix} + \begin{pmatrix}2\\1\end{pmatrix}$

$= \frac{1}{\sqrt{5}}\begin{pmatrix}-7\\4\end{pmatrix} + \begin{pmatrix}2\\1\end{pmatrix}$

$= \frac{1}{\sqrt{5}}\begin{pmatrix}-7+2\sqrt{5}\\4+\sqrt{5}\end{pmatrix}$

b) ${}_E K_O(v) = O v + \theta$

$= \frac{1}{\sqrt{5}}\begin{pmatrix}2 & -1\\1 & 2\end{pmatrix} v + \begin{pmatrix}2\\1\end{pmatrix}$

c) ${}_O K_E(v) = O^{-1}(v - \theta) = O^{-1} v - O^{-1}\theta$

$= \frac{1}{\sqrt{5}}\begin{pmatrix}2 & 1\\-1 & 2\end{pmatrix} v - \frac{1}{\sqrt{5}}\begin{pmatrix}2 & 1\\-1 & 2\end{pmatrix}\begin{pmatrix}2\\1\end{pmatrix}$

$= \frac{1}{\sqrt{5}}\begin{pmatrix}2 & 1\\-1 & 2\end{pmatrix} v - \frac{1}{\sqrt{5}}\begin{pmatrix}5\\0\end{pmatrix}$